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COMPARISON OF GENERALIZED NONLINEAR HEIGHT-DIAMETER MODELS FOR *PINUS HALEPENSIS* MILL. AND *QUERCUS CERRIS* L. IN SICILY (SOUTHERN ITALY)

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*Various models of the relationship between tree stem height (*h*) and tree stem diameter at breast height (*dbh*) generalized by forest stand characteristics (stand dominant height,* H_{dom}*, and stand dominant* dbh*,* D_{dom}*) were compared to assess their prediction potential for Pinus halepensis and Quercus cerris in Sicily (Southern Italy). The models were designed to be used in a large scale level with a wide range of forest conditions. Comparison of the models was carried out by studying the bias, the root mean square error, the Akaike's information criterion and the Pearson correlation coefficient between observed and predicted value. Among those compared, the most suitable model for both species proved to be:*

$$\hat{h}_i = \frac{(1-e^{b_1 d_i})^{b_2}}{(1-e^{b_1 D_{dom}})^{b_2}}\ H_{dom}$$

Key words: height-dbh equation; non linear model; large-scale level; *Pinus halepensis* Mill.; *Quercus cerris* L.

Parole chiave: equazione altezza-diametro; modelli non lineari; scala vasta; *Pinus halepensis* Mill.; *Quercus cerris* L.

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1. INTRODUCTION

Tree stem diameter at breast height (dbh) and height are main dendrometric attributes measured in forest inventory. Measuring dbh is easier and more reliable than measuring tree height. As a consequence, in forest inventories full-tree callipering is carried out on the inventoried stand, or on sample plots within the stand, while height is measured only for a sample of trees within the stand, or within each sample plot. To predict height for all the trees in the stand (or in each sample plot) statistical models are fitted to establish the relationship between these two attributes. Usually the fitting is carried out at stand or plot level.

The height-dbh relationship allows for the assessment of tree volume as well as the description of stand and its development over time (CURTIS, 1967). As a result, the relationship between tree stem height (*h*) and dbh (*d*) is one of the most studied in forestry. Literature height-diameter relationship is copious (e.g. CURTIS, 1967; WYKOFF *et al.*, 1982, LARSEN and HANN, 1987; WANG and HANN, 1988; HUANG *et al.*, 1992; MOORE *et al.*, 1996; ZHANG, 1997; PENG, 1999; FANG and BAILEY, 1998; JAYARAMAN and ZAKRZEWSKI, 2001; ZHANG, 2002; TEMESGEN *et al.*, 2007).

Although stem dbh is a suitable predictor of stem height, the relationship between these two dendrometrical attributes varies from one stand to another (CALAMA and MONTERO, 2004) and may be not constant over time even within the same stand (CURTIS, 1967). There are also other factors determining the relationship. The most obvious among these factors is growing space and stand conditions (SHARMA and ZHANG, 2004): for a particular height, trees that grow in high density stands tend to have smaller dbh than those growing in less dense stands, because of greater competition

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among individuals (LOPEZ SANCHEZ *et al.*, 2003; CALAMA and MONTERO, 2004). These facts highlight that stand-level attributes are required for providing generalized height-dbh equations able to predict individual tree height for forest stands on large territories (e.g. at national or subnational) levels maintaining projections within reasonable biological limits see TEMESGEN and GADOW, 2004.

Several approaches have been exploited in the development of generalized height-dbh models. For instance, HARRISON *et al.* (1986) included stand dominant height in their height-dbh model. SOARES and TOMÉ (2002) used stand dominant height, maximum dbh, and density as predictor variables in addition to dbh. LOPEZ SANCHEZ *et al.* (2003) and EERIKAINEN (2003) used stand dominant height, dominant diameter, density, and age information in their models to improve model accuracy. Similarly, ZAKRZEWSKI and BELLA (1988) exploited quadratic mean dbh and the height of tree with quadratic mean dbh to increase model efficiency.

The aim of this work is to develop and test generalized *h-d* models by taking into account stand dominant height (H_{dom}) and stand dominant dbh (D_{dom}) as additional variables which may significantly characterize this relationship. H_{dom} and D_{dom} are usually assessed by forest inventories.

2. DATA

Data used in this study were collected during the forest inventory of the Sicily Region (Southern Italy). The sampling design adopted for performing the regional forest inventory was linked to the National Inventory of Forest and of Forest Carbon Sinks (see www.infc.it). Probability sampling provided consistent data, most likely covering the entire range of tree and stand attributes. Data collected in robust inventory sampling provide a vast source of data that could be harnessed regularly and effectively to develop broadscale forest models. A detailed description of sampling design can be found in HOFMANN *et al.* (2011) while the protocol adopted for measuring tree height and dbh at plot level is accounted in INFC (2006). For each sample plot, the tree level information concerns tree status (live or dead), origin (natural or planted), species and dbh for all the trees, and tree height for a subset of trees for a maximum of ten trees selected per plot. The average dbh of the three trees with largest dbh in the 530 m^2 wide sample plot is conventionally considered as D_{dom}, while H_{dom} is conventionally computed as the average height of the same trees. In this note, *d* and D_{dom} are expressed in cm, *h* and H_{dom} in m.

We analysed the data from the most widespread species in the region, one conifer (*Pinus halepensis*) and one broadleaf (*Quercus cerris*). The available dataset was constituted by 108 plots from stands with prevailing *Pinus halepensis* and 65 plots from stands with prevailing *Quercus cerris*. Calibration datasets for model fitting were randomly selected as 70% of plots for each species. The remaining 30% of plot for each species were used as validation datasets. The mean, maximum and minimum values and standard deviation of dendrometric variables for both the calibration and the validation datasets are reported in Tables 1 and 2.

3. METHODS

The compared *h-d* models were developed from the basic Chapman-Richards nonlinear one, which has been frequently used for modeling the height-dbh relationship (MEYER, 1940; FARR *et al.*, 1989)

$$\hat{h}_i = a\,(1 - e^{bd_i})^c$$

where is the predicted stem height for the *i*-th tree, d_i is the measured stem dbh of the *i*-th tree, and a, b and c are the asymptote, rate and shape coefficients respectively.

INFC (2009) proposed the following generalization of this model in the form

$$\hat{h} = (b_1 + b_2 H_{dom})\,(1- e^{(b_3+b_4 H_{dom}) d_i})^{(b_5+b_6 H_{dom})} \quad (1)$$

In this study the generalization of the Chapman-Richards model was carried out by also including $D_{dom}$to better account for

Table 1 – Summary statistics of the *Pinus halepensis* dataset.

Variable	Mean	Max	Min	Dev. standard
		Calibration dataset (N = 428)		
h	10.75	23.30	1.30	4.21
H_{dom}	11.49	19.77	1.72	3.86
d	23.85	56.00	4.50	11.59
D_{dom}	28.99	53.50	6.25	10.94
		Validation dataset (N =166)		
h	11.68	23.70	3.20	4.26
H_{dom}	13.03	20.10	4.47	3.82
d	27.58	54.50	6.00	11.74
D_{dom}	32.96	53.00	10.83	10.81

Table 2 – Summary statistics of the *Quercus cerris* dataset *Calibration dataset (N= 235)*

Variable	Mean	Max	Min	Dev. standard
		Calibration dataset (N= 235)		
h	12.16	20.50	4.80	3.75
H_{dom}	13.07	19.53	5.77	3.46
d	24.90	80.00	8.00	11.79
D_{dom}	30.50	58.67	10.33	9.67
		Validation dataset (N= 117)		
h	12.96	22.90	3.20	4.25
H_{dom}	14.21	21.03	8.43	3.59
d	24.38	105.00	6.00	16.21
D_{dom}	31.98	94.33	18.83	12.94

the stand structure. Not more than four coefficients were allowed in the tested models, in order to keep them parsimonious and to likely prevent multicollinearity and eventual non-convergence in the nonlinear estimation of the coefficients. Moreover, the models were constrained so that when $d_i = D_{dom}$ then $h_i = H_{dom}$. Thus, the following models were tested:

$$\hat{h}_i = \frac{(1-e^{b_1 d_i})^{b_2}}{(1-e^{b_1 D_{dom}})^{b_2}} H_{dom} \tag{2}$$

$$\hat{h}_i = \frac{(1-e^{b_1 d_i})^{(b_2+b_3 H_{dom})}}{(1-e^{b_1 D_{dom}})^{(b_2+b_3 H_{dom})}} H_{dom} \tag{3}$$

$$\hat{h}_i = \frac{[1-e^{(b_1+b_2 H_{dom}) d_i}]^{(b_3+b_4 H_{dom})}}{[1-e^{(b_1+b_2 H_{dom}) D_{dom}}]^{(b_2+b_3 H_{dom})}} H_{dom} \tag{4}$$

From a mathematical point of view, for all models the domain is defined for all real numbers and no global maximum and minimum points are found. H_{dom} constitutes the horizontal asymptote for models [2] to [4]. There are no vertical and oblique asymptotes for each models.

Models were fitted by the calibration dataset. Model coefficients were estimated by the Levenberg-Marquardt algorithm in ordinary nonlinear least squares regression with SPSS software. No outlier elimination was carried out, given the methodological comparative purposes of this work.

The predictive potential of the obtained *h-d* models were independently assessed by the validation dataset, considering four different criteria as references: bias, root mean square error (RMSE), Akaike information criterion (AIC), Pearson correlation coefficient between observed and predicted values (r).

4. Results and discussion

Tables 3 and 4 show the regression coefficients and relative standard error for models from [1] to [4] obtained from the calibration dataset.

For both species, all the *t*-statistics for the coefficients of the model [2] are significant at the 0.05 level. For all the other models, with the exception of *Quercus cerris* in model [3], at least one coefficient results not significant at the 0.05 probability level. The range of standard error of estimate among the compared models is relatively modest: from 1.54 up to 1.60 m for *Pinus halepensis* and from 1.58 up to 1.70 m for *Quercus cerris*.

The validation dataset has been used to assess prediction performance of the different compared models. Tables 5 and 6 displays the fit statistics (bias, RMSE, AIC, r) which vary significantly across models for a given species. As concerns *Quercus cerris*, the models [2] and [4] have the best and poorest fits, respectively. The results are upside down for *Pinus halepensis*: model [2] has the poorest fit statistics while model [4] generally performs better than the remaining models. Models [1]

Table 3 – Regression coefficients and the standard errors of estimate (SEE, expressed in m) for the developed models for *Pinus halepensis.*

Model	b_1	b_2	b_3	b_4	b_5	b_6	*SEE*
[1]	0.372	0.983*	-0.162*	0.002	-0.089	0.111*	1.600
[2]	-0.057*	0.631*					1.546
[3]	-0.072*	0.402	0.029				1.545
[4]	-0.052	-0.001	0.295	0.036			1.546

* the asymptotic t-statistic for the regression coefficient is significant at 0.05 probability level.

Table 4 – Regression coefficients and the standard errors of estimate (SEE, expressed in m) for the developed models for *Quercus cerris.*

Model	b_1	b_2	b_3	b_4	b_5	b_6	*SEE*
[1]	0.219	1.012*	-0.221*	0.005	-0.533	0.171*	1.699
[2]	-0.046*	0.650*					1.705
[3]	-0.071*	-0.749*	0.114*				1.586
[4]	-0.169*	0.005	-0.240	0.090*			1.576

* the asymptotic t-statistic for the regression coefficient is significant at 0.05 probability level.

Table 5 – Model performances assessed by the validation dataset for *Pinus halepensis.*

Model	*Bias (m)*	*RMSE (m)*	*r*	*AIC*
[1]	-0.536	1.797	0.919	200.516
[2]	-0.586	1.876	0.909	210.933
[3]	-0.569	1.786	0.919	195.462
[4]	-0.042	1.532	0,935	145.665

and [3] produced similar fit statistics. As mentioned, all coefficients of model [3] are significantly different from zero at least for *Quercus cerris*. The inclusion of an additional variable lowers bias and RMSE but it increases r and AIC values. For this reason the model is not considered best approximating model for this species. Model [2] had the smallest variation in RMSE and r across species while model [4] showed the same results for bias and AIC.

Among the two best fitted models (i.e., the model [2] for *Quercus cerris* and the model [4] for *Pinus halepensis*), the model [4] has resulted non significant *t*-statistics for at least two parameters of the model. Since the performance gain of this model over model [2] is not substantial for *Pinus halepensis* as proved by the validation test, the model [2] may be preferred even for this species. The plot of residuals vs. predicted heights and the plot of predicted vs. observed heights show that model [2] appropriately enough fits the validation data for both the considered species.

The model [2] provides better performance especially in *Quercus cerris* plots, albeit an overestimation of height for the lower diameter class can be observed. Validation data highlight obvious outliers, in particular for the lower diameter class: as mentioned, given the comparative meaning of the present testing, the outliers were purposely not removed from the dataset.

5. Conclusions

Tree stem height is a relevant attribute for forest inventory and even to characterize tree, stand, and site conditions. However, measuring tree height is less reliably and more expensive than measuring tree dbh, so it is usually limited to a sample of trees. By relating height and diameter, non-measured heights can be predicted by suitable height-diameter functions.

The inclusion of stand dominant height and stand dominant dbh into the height-diameter function allows to generalize the prediction relatively overlarge areas. Dominant diameter and dominant height are stand measures easily obtained and available so the use of the height-diameter model with these attributes is suggested to improve the accuracany of height prediction.

In this study, the model is developed from a robust inventory dataset representing the entire conditions of Sicilian forests for the selected species. For both conifers and hardwoods, the fit statistics and coefficients estimation indicated that the model [2] is the most suitable for predicting height-diameter relationships in Sicily. The suggested model allows the natural variability in height within diameter class to be mimicked and therefore provides realistic height prediction at stand level. The height-diameter model developed in this study maintains projections within reasonable biological limits and in general, the presented *h-d* model seems to hold an appropriate level of reliability. However the models should be evaluated and, if necessary, revisited or calibrated when they are applied in different regions.

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RIASSUNTO

Comparazione tra modelli ipso-diametrici generalizzati per Pinus halepensis *Mill. e* Quercus cerris *L. in Sicilia (Sud Italia)*

Modelli che esprimono la relazione esistente tra altezza e diametro sono stati generalizzati attraverso le principali caratteristiche del popolamento forestale (altezza dominante, H_{dom}, e diametro dominante, D_{dom}). I modelli sono stati sviluppati per essere utilizzati su larga scala e in presenza di diverse condizioni forestali. Le analisi sono state qui condotte per *Pinus halepensis* e per *Quercus cerris* in Sicilia. Il confronto tra i modelli considerati è stato effettuato studiando la distorsione di stima, l'errore quadratico medio, il criterio di Akaike e il coefficiente di correlazione di Pearson tra i valori osservati e predetti. Per entrambe le specie considerate l'equazione che meglio descrive la relazione altezza-diametro è:

$$\hat{h}_i = \frac{(1-e^{b_1 d_i})^{b_2}}{(1-e^{b_1 D_{dom}})^{b_2}}\ H_{dom}$$

REFERENCES

Calama R., Montero G., 2004 – *Interregional nonlinear height–diameter model with random coefficients for stone pine in Spain.* Canadian Journal of Forest Research, 34: 150-163. http://dx.doi.org/10.1139/x03-199

Corona P., 2000 – *Introduzione al rilevamento campionario delle risorse forestali.* Edizioni CUSL, Firenze.

Curtis R.O., 1967 – *Height-diameter and height-diameter-age equations for second-growth Douglas-fir.* Forest Science, 13: 365-375.

Eerikainen K., 2003 – *Predicting the height-diameter pattern of planted* Pinus kesiya *stands in Zambia and Zimbabwe.* Forest Ecology and Management, 175: 355-366. http://dx.doi.org/10.1016/S0378-1127(02)00138-X

Fang Z., Bailey R.L., 1998 – *Height-diameter models for tropical forests on Hainan Island in southern China.* Forest Ecology and Management, 110: 315-327. http://dx.doi.org/10.1016/S0378-1127(98)00297-7

Farr W.A., DeMars D.J., Dealy J.E., 1989 – *Height and crown width related to diameter for open-grown western hemlock and Sitka spruce.* Canadian Journal of Forest Research, 19: 1203-1207. http://dx.doi.org/10.1139/x89-181

Harrison W., Burk T., Beck D., 1986 – *Individual tree basal area increment and total height equations for Appalachian mixed hardwoods after thinning.* Southern Journal of Applied Forestry, 10: 99-104.

Hoffmann A., Cibella R., Bertani R., Miozzo M., Fantoni I., Luppi S., (a cura di), 2011 – *Strumenti conoscitivi per la gestione delle risorse forestali della Sicilia. Sistema informativo Forestale.* Assessorato Territorio e Ambiente. Regione Siciliana 208 p.

Huang S., Titus S.J., Wiens D.P., 1992 – *Comparison of nonlinear height-diameter functions for major Alberta tree species.* Canadian Journal of Forest Research, 22: 1297-1304. http://dx.doi.org/10.1139/x92-172

Jayaraman K., Zakrzewski W.T., 2001 – *Practical approaches to calibrating height-diameter relationships for natural sugar maple stands in Ontario.* Forest Ecology and Management, 148: 169-177. http://dx.doi.org/10.1016/S0378-1127(00)00534-X

INFC, 2006 – *Procedure di posizionamento e di rilievo degli attributi di terza fase con istruzione per l'impiego degli applicativi NAV3 e RAVS3.* Inventario Nazionale delle Foreste e dei Serbatoi Forestali di Carbonio. MiPAA - Corpo Forestale dello Stato, CRA-ISAFA, Trento.

INFC, 2009 – *I caratteri quantitativi* 2005 *- parte 1, vers. 2.* Autori P. Gasparini, F. De Natale, L. Di Cosmo, C. Gagliano, I. Salvadori, G. Tabacchi e V. Tosi. Inventario

Nazionale delle Foreste e dei Serbatoi Forestali di Carbonio. MiPAAF - Ispettorato Generale Corpo Forestale dello Stato, CRA-MPF, Trento.

LARSEN D.R., HANN D.W., 1987 – *Height-diameter equations for seventeen tree species in southwest Oregon.* Oregon State University. Forest Research Laboratory, Research Paper.

MEYER H.A., 1940 – *A mathematical expression for height curves*. Journal of Forestry, 38: 415±420.

MOORE J.A., ZHANG L., STUCK D., 1996 – *Height-diameter equations for ten tree species in the Inland Northwest.* Western Journal of Applied Forestry, 11: 132-137.

LÓPEZ SÁNCHEZ C.A., GORGOSO VARELA J., CASTEDO DORADO F., ROJO ALBORECA A., RODRÍGUEZ SOALLEIRO R., ÁLVAREZ GONZÁLEZ J.G., SÁNCHEZ RODRÍGUEZ F., 2003 – *A height-diameter model for* Pinus radiata *D. Don in Galicia (Northwest Spain)*. Annals of Forest Science, 60: 237-245. http://dx.doi.org/10.1051/forest:2003015

PENG C., 1999 – *Nonlinear height-diameter models for nine boreal forest tree species in Ontario*. Ministry of Natur. Resour. Ontario Forest Research Institute, OFRIR ep. 155 .

SOARES P., TOMÉ M., 2002 – *Height-diameter equation for first rotation eucalypt plantations in Portugal.* Forest Ecology Management, 166: 99-109.

SHARMA M., ZHANG S.Y., 2004 – *Height-diameter models using stand characteristics for* Pinus banksiana *and* Picea mariana*.* Scandinavian Journal of Forest Research, 19: 442-451. http://dx.doi.org/10.1080/02827580410030163

TEMESGEN H., GADOW K.V., 2004 – *Generalized height-diameter model - an application for major tree species in complex stands of interior British Columbia.* European Journal of Forest Research, 123: 45-51. http://dx.doi.org/10.1007/s10342-004-0020-z

TEMESGEN H., HANN D.W., MONLEON V.J., 2007 – *Regional Height - DiameterEquations for Major Tree Species of Southwest Oregon.* Western Journal of Applied Forestry, 22 (3): 213-219.

WANG C.H., HANN D.W., 1988 – *Height-diameter equations for sixteen tree species in the central western Willamette valley of Oregon*. Oregon State University. Forest Research Laboratory, Research Paper.

WYKOFF W.R., CROOKSTON N.L., STAGE A.R., 1982 – *User's guide to the stand prognosis model.* USDA Forest Service. General Technical Report. INT-133.

ZAKRZEWSKI W.T., BELLA I.E., 1988 – *Two new height models for volume estimation of lodgepole pine stands.* Canadian Journal of Forest Research, 18: 195-201. http://dx.doi.org/10.1139/x88-030

ZHANG L., 1997 – *Cross-validation of nonlinear growth functions for modeling tree height-diameter distributions.* Annals of Botany, 79: 251-257. http://dx.doi.org/10.1006/anbo.1996.0334

ZHANG L., PENG C., HUANG S., ZHOU X., 2002 – *Development and evaluation of ecoregion-based jack pine height - diameter models for Ontario*. Forestry Chronicle, 78: 530-538.